Probabilistic Assessment of Commercial Design Guides for Steep Reinforced Slopes: Implications for Design

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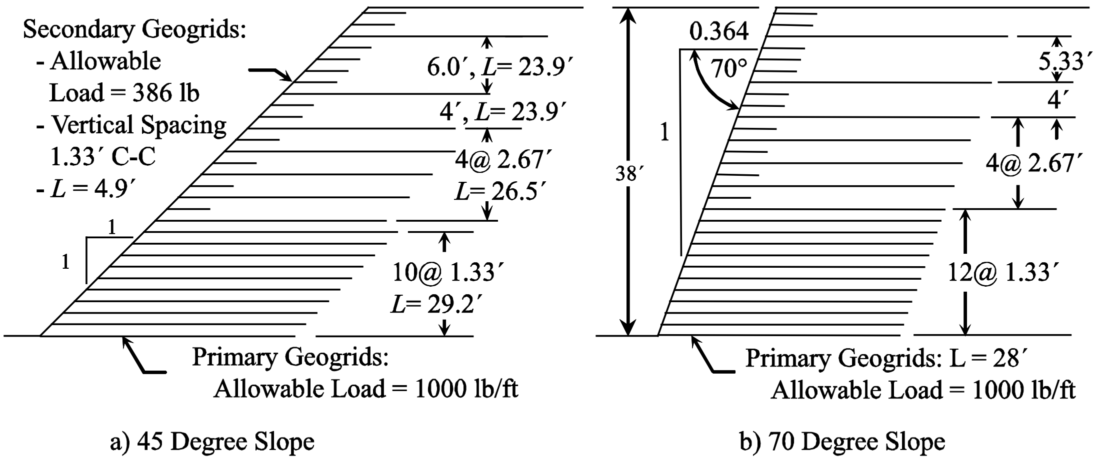
ABSTRACT

Deterministic procedures are still the predominate methods used in reinforced slope design. These procedures generally use limit equilibrium methods based on accepted factors of safety. This paper presents a method of probabilistic analysis which is easily adapted to existing deterministic methods. The method is used to analyze the reliability of a two steep reinforced slopes designed using commercially published guidelines. The reliability of internal failure modes (passing within the reinforced zone) and composite failure modes (passing mostly outside of the reinforced zone) are computed and compared. The results indicate that internal and composite failure modes may be nearly equally likely leading to a total probability of failure of twice that of either mode. The paper present a design process to ensure that the probability of failure is controlled by composite failure modes and the reliability of internal modes is sufficiently high that they do not significantly reduce the total reliability of the slope.

SLOPE DESIGNS ANALYZED

This paper exams two reinforced slopes designed using typical deterministic methods and analyzes these designs using probabilistic methods. The slopes are 45° and 70° slopes reinforced with geogrids. Both designs consist of uniform cohesionless soil both within and below the slopes. The slopes were designed using guidelines published by Tensar (1988). These design guidelines were developed with a two wedge failure surface satisfying only force equilibrium, Schmertmann et al (1987). For a uniform soil with a given friction angle, ϕ, and slope angle, β, the key design parameters are: the length of the reinforcing elements, L, chosen so that composite failure modes have a satisfactory factor of safety, and the spacing, S, and allowable reinforcement force, R_a, selected so that internal failure modes have a satisfactory factor of safety. The two slopes designed using the Tensar (1988) guidelines are shown in Figure 1.



a) 45 Degree Slope b) 70 Degree Slope

Figure 1. Cross-sections of two reinforced slopes analyzed showing length and spacing of geogrid reinforcement

DETERMINISTIC ANALYSES

After the slopes were designed using the Tensar (1988) guidelines they were reanalyzed using UTEXAS3 (Wright 1991). This analysis used a full equilibrium method with circular failure surfaces. This allowed failures extending into base materials below the slopes to be investigated. It also allowed investigation of the relationship between allowable reinforcement force and reinforcement length and how these variables affect the location of the critical surface.

Nineteen distinct failure modes were investigated: 18 internal modes passing through the reinforced portion of each slope, and one composite mode passing predominately outside the reinforced portion of each slope but intersecting a few reinforcing elements. The failure modes were numbered 0 (zero) for the composite mode and 1 through 18 for the internal modes, the number indicated the lowest reinforcing elements through which the failure surface passed. Typical failure modes are shown in Figure 2.

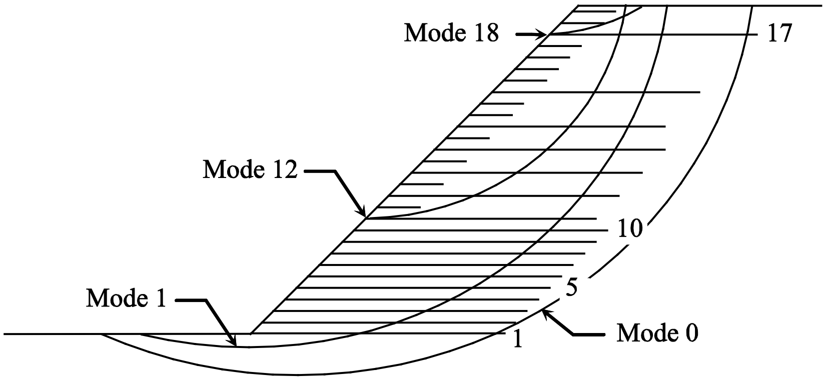


Figure 2. Failure modes of 45 degree showing numbering scheme

For each failure mode a search for the critical slip surface was performed to determine the local minimum factor of safety for that mode. The computed factors of safety for each mode are shown in Figure 3. Figure 3 shows that the Tensar (1988) design guidelines generate slope designs where the composite failure mode controls and the critical surface passes predominately below and behind the reinforced portion of the slope. Figure 3 also shows that the internal failure modes have a relatively uniform factor of safety. Both these observations are in keeping with the original intent of these design guidelines (Schmertmann et al, 1987).

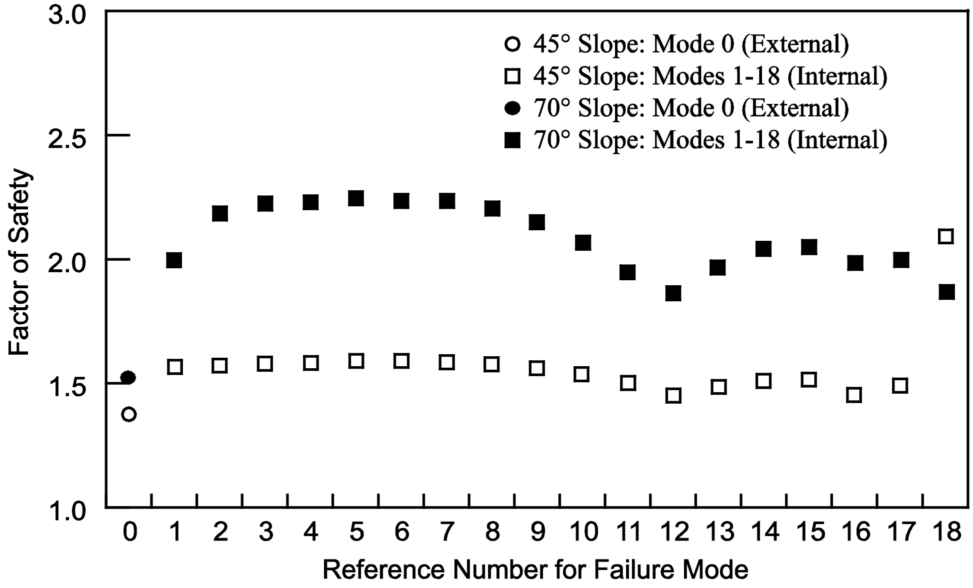


Figure 3. Comparison of the factors of safety computed for each failure mode

A second series of analyses was done to investigate the relationship between allowable reinforcement force and reinforcement length. In this series of analyses the vertical spacing of geogrids was kept the same, but the reinforcement length and allowable reinforcement force were varied. A large number of combinations of reinforcement length and force were analyzed; for each combination the minimum factor of safety was computed and the location of the critical circle identified. Figure 4, show the results of this analysis for the 45° slope. Figure 4 shows that for relatively long or weak reinforcement (the lower shaded portion of the figure) the failure mode is internal and the factor of safety is a function of only the reinforcement force. Figure 4 also shows that for relatively strong or short reinforcement, the failure mode is composite and the factor of safety is governed by the reinforcement length. Note that the transition from internal to composite failure mode is quite abrupt. It is also worth noting that the Tensar (1998) design guides have a composite failure mode close to the point where the failure surface transition from composite to internal.

PROBABILISTIC ANALYSES

Using the deterministic analyses as a starting point, the reliability index, β, was computed for each of the 19 failure modes. These analyses used two random variables: the shear strength or the soil, s, in the form of $s = \tan(\phi)$, and the reinforcement force, *R*. The reinforcement length and spacing were assumed to have no significant variance. The limit state function was taken to be

$$g\left(s,R\right)=1-F(s,R) \tag{1}$$

Where, *F* is the factor of safety computed using standard limit equilibrium methods.

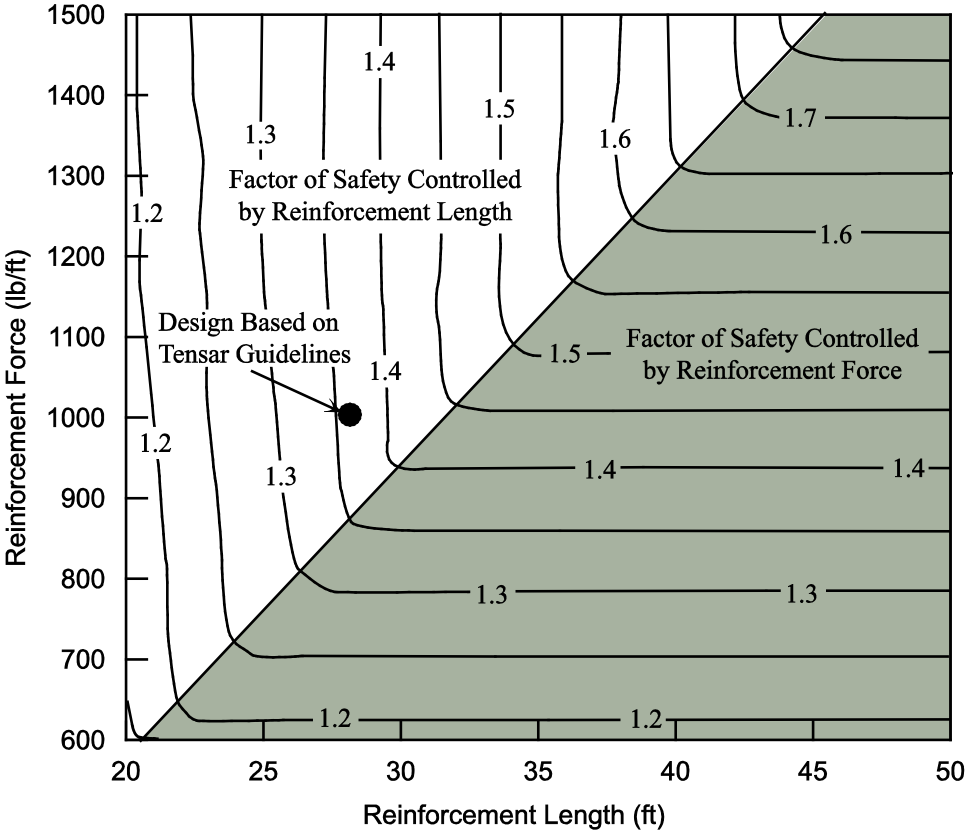


Figure 4. Factor of safety as a function of reinforcement length and force for the 45 degree slope

The mean-value first-order reliability index, β_{MV}, because of simple form

$$\beta_{MV}=\frac{1-F(\mu_s,\mu_R)}{\sqrt{\frac{\partial F}{\partial s}\sigma_s{}^2+\frac{\partial F}{\partial R}\sigma_R{}^2}} \tag{2}$$

Where μ_s and μ_R are the mean values of the soil shear strength and reinforcement force and σ_s and σ_R are the standard deviations of the soil shear strength and reinforcement force (Ang and Tang, 1984, Madsen et al 1986). UTEXAS3 was used to find the minimum factor of safety for a given failure mode which was then $F(\mu_s, \mu_R)$. Four more computations with UTEXAS3 were then needed to numerically estimate the partial derivatives in Equation 2. The mean-value first-order reliability index was then computed knowing the standard deviations of the shear strength and reinforcement force (Kitch 1994).

Figure 5 shows the mean-value reliability index calculated for each of the 18 failure modes assuming a coefficient of variation of 0.1 for shear strength and 0.2 for reinforcement force for the 45° slope. From this figure it can be seen that mode 12 has the highest probability of failure (lowest *β*) for internal failure modes. Mode 0 represents the composite failure mode with the highest probability of failure.

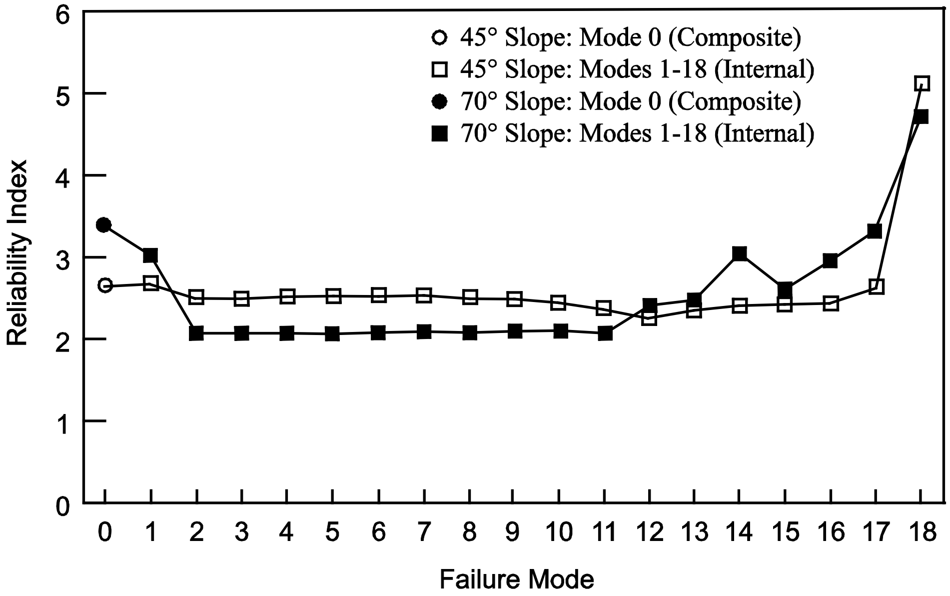


Figure 5. MFORM reliability indices for failure modes in each slope

UNCERTAINTY IN SOIL STRENGTH AND REINFORCEMENT FORCE

One advantage of the mean-value first-order reliability method is that it is easy to determine how variance of the random variables affects reliability. Once the mean value of the factor of safety, *F*, and the partial derivatives of *F* with respect to shear strength and reinforcement force are computed, the reliability index can be computed for any combinations of σ_S and σ_R without any further limit equilibrium computations, using Equation 2.

Published literature on the variability of shear strength of cohesionless soils and mobilized reinforcement force in reinforced slopes was reviewed (Kitch, 1994). Based on this review the values shown in Table 1 were determined. It should be noted that the estimates are for variability of a single known cohesionless soil. They do not include the uncertainty of the existence of an unidentified soil layer. These values are appropriate for a built up slope where the embankment soil is known and has been tested. They would not be appropriate for the remediation of an existing slope where there is a possibility of unidentified weak layers existing beneath or behind the reinforced slope. In Table 1, inherent variability considers only the variability of the material properties; model error considers uncertainties in how properties are determined and how they are modeled in the analysis process.

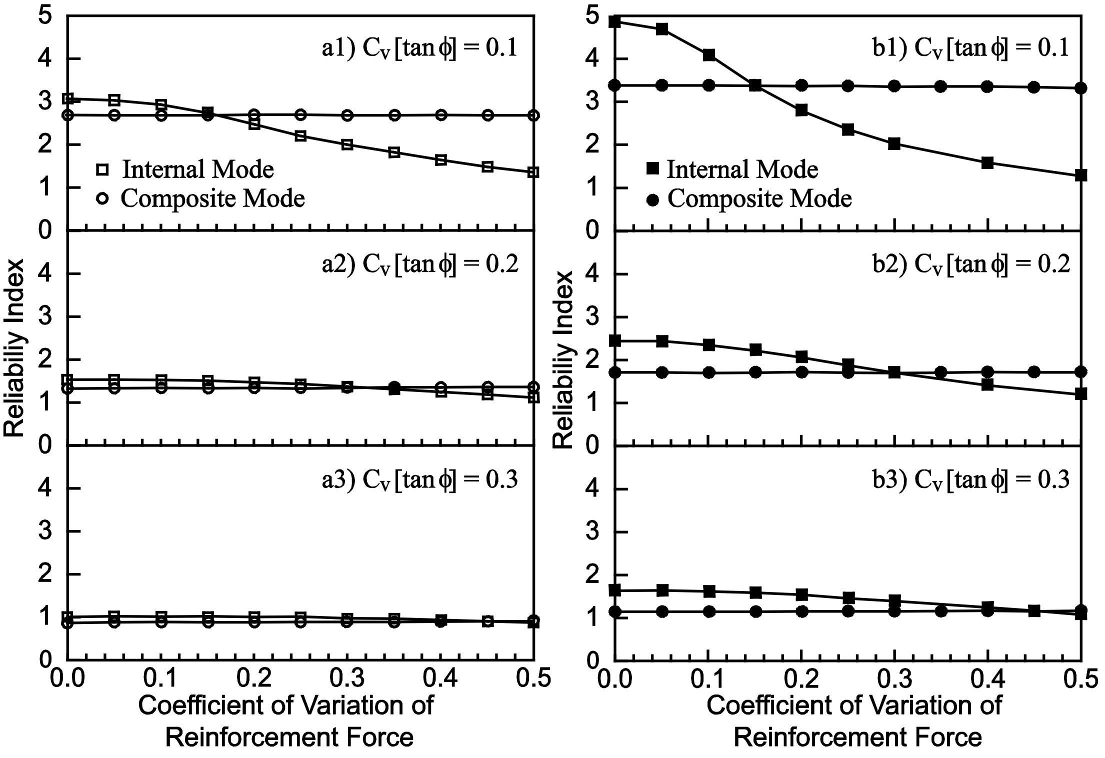
Table 1. Variability Estimates for Shear Strength and Reinforcement Force*

	Coefficient of Variation	
Variable	Inherent variability	Model error
Spatial average shear strength	0.10	0.15 – 0.18
Reinforcement Force	0.10	0.25

* From Kitch (1994)

From the data in Table 1, it is estimated that the coefficient of variation or shear strength could range from 0.10 to 0.20 (=$\sqrt{0.10^2 + 0.18^2}$) depending on whether or not model error is included. Similarly the coefficient of variation of reinforcement force could range from 0.10 to 0.27 (= $\sqrt{0.10^2 + 0.25^2}$). Using these ranges of variability, a sensitivity study was conducted to determine how the relative variability of soil strength and reinforcement force affected the probability of internal versus composite failure modes.

For the sensitivity study, the reliability of the critical internal failure mode was compared with that of the composite failure mode for coefficients of variation of the shear strength of 0.1, 0.2 and 0.3 and coefficients of variation of the reinforcement force ranged from zero to 0.5. The comparison of the reliabilities for internal and composite failure modes as a function of the coefficients of variation is shown in Figure 6 for both slopes.



a) 45 Degree Slope b) 70 Degree Slope

Figure 6. Comparison of reliability index of internal and composite failure modes as a function of variability of reinforcement force and soil shear strength.

If we consider only inherent variability of the shear strength and the reinforcement force, then coefficients of each variable are approximately 0.10 (Table 1) and for both the 45° and 70° slope the most likely failure mode is internal. However, if we include model error then the coefficients of variation increase to 0.20 for shear strength and 0.27 for reinforcement force. In this case, not only do the reliabilities of the slopes decrease both the probability of an internal and composite failure modes are nearly equal as can be seen in Figures 6a and 6c.

IMPLICATIONS FOR ANALYSIS AND DESIGN

The analysis presented shows that reinforced slopes designed using the Tensar (1998) guidelines have their critical failure surface with the minimum factor of safety is a surface falling primarily outside of the reinforced portion of the slope. Newer procedures (Elias et al, 2000 and Tensar, 2003) generate similar designs. However, the probabilistic analysis indicates that for typical levels of uncertainty in the reinforcement force and soil shear strength, the internal and composite failures are nearly equally probable. Since the two modes are essentially independent the total probability of failure is the sum of the probabilities of failure of each. In the case where the probabilities of failure of each mode are equal the total probability is twice the maximum probability of failure of either mode. If the slope is designed such that the probability of failure of internal modes is significantly lower than the probability of composite modes then the probability of failure of the slope will be equal to the probability of failure of the composite mode.

The designer has little control over the variability of either soil shear strength or the reinforcement force. The mean soil shear strength is also limited by the availability of soils at or near the site. In this case the key remaining design variables are the reinforcement strength and spacing which control the mean reinforcement force, and the reinforcement length which will control the location of the composite failure mode and thereby the mean factor of safety of the composite failure mode.

If the coefficients of variation are fixed, the reliability of internal failure modes will be controlled by the mean reinforcement strength which can be increased by either decreasing the spacing of reinforcement or increasing the strength of individual reinforcing elements. In either case this is relatively inexpensive. The reliability of the composite failure mode will then be controlled by the mean factor of safety of the composite failure mode, which, in turn, will be controlled by the length of the reinforcement. Increasing the reinforcement length has a significant increase in the earthwork costs which will, in general, exceed the costs of any additional reinforcing material. Therefore the key design parameter for most slopes will be length of reinforcement used. Since the marginal costs of increasing reinforcement within the reinforced zone of the slope is low, it should always be possible to design a slope where the most probably failure surface is a composite surface. And the probability of failure will be controlled by the reinforcement length and the mean and coefficient of variation of soil strength.

Combining MFORM analyses with traditional limit equilibrium stability analyses is a useful in designing reinforced slopes. The process should start by estimating the mean value of the soil strength and the variability of the soil strength and reinforcement force. The coefficients of variation should be selected conservatively. With this information, the analysis should proceed by determining the length of reinforcement required to provide an adequate reliability for the composite failure mode. Then the spacing and strength of reinforcement can be chosen so that the probability of failure of an internal failure is well below that of a composite failure.

It is relatively simple to expand a typical limit equilibrium analysis into an MFORM probabilistic analysis using any limit equilibrium slope stability analysis program that accommodates reinforcement. The process starts by locating the critical slip surface and computing the associated factor of safety using the mean value of the random variables, (in this case shear strength and reinforcement force). Once the critical surface is found, 2 more limit equilibrium computations must be performed for each random variable in order to compute the partial derivatives needed to compute mean-value reliability index (Equation 2). Once the partial derivatives have been computed, the reliability index can be computed for any combination of coefficients of variation of the random variables without further limit equilibrium computations. This makes MFORM a simple and powerful tool to investigate how changes in variability affect the reliability of various slope designs. No special probabilistic software is needed.

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